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Multiple stable patterns in a balanced bistable equation with heterogeneous environments

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1 Introduction and Main Result

There are several results on the studies of solutions to the following equation with a balanced bistable nonlinearity:

$$\epsilon^2 \Delta u + h(x)^2 (a(x)^2 - u^2) u = 0, \quad x \in \Omega, \quad \frac{\partial u}{\partial n} = 0, \quad x \in \partial\Omega,$$

where Ω is a bounded domain in $\mathbf{R}^n, n \geq 1$ with smooth boundary, $\epsilon > 0$ is a parameter, and $h(x)$ and $a(x)$ are positive functions on Ω. Solutions u of the boundary value problem above is corresponding to critical points of the functional

$$J(u) = \frac{1}{2}\epsilon^2 \int_\Omega |\nabla u(x)|^2 \, dx + \frac{1}{4}\int_\Omega h(x)^2 (a(x)^2 - u(x)^2)^2 \, dx$$

on $H^1(\Omega)$. The global minimizer $u(x)$ of $J(u)$ on $H^1(\Omega)$ has an asymptotic behavior $u(x) \to a(x)$ (or $u(x) \to -a(x)$) as $\epsilon \to 0$. In general, to find a nontrivial local minimizer $u(x)$ with inner transition layers is a delicate problem.

If the dimension is one, there are several results. Let $\Omega = (0,1)$. When $h(x) \equiv 1$, Nakashima [8] proved by using a delicate construction of a subsolution and a super-solution that if $a \in C^2[0,1]$ takes a nondegenerate local minimum at $x_0 \in (0,1)$, then there exists a stable solution which has the asymptotic behavior $u_\epsilon(x) \sim -a(x)$ on $(0, x_0)$ and $u_\epsilon(x) \sim a(x)$ on $(x_0, 1)$ as $\epsilon \to 0$. Later, Matsuzawa [7] extended her result in a degenerate setting. On the other hand, when $a(x) \equiv 1$, Nakashima [9] also constructed a stable solution which has an inner transition layer near a local minimal point of $h(x)$ and studied the location of inner transition layers of solutions in details.

Furthermore, Nakashima-Tanaka [10] constructed solutions with multi-transition layers systematically by using variational methods.

For the studies in the higher dimensional case and $a(x) = 1$, we refer to [3], [6], [11], [12]. In these previous results, the effect of domain geometry or the effect of $h(x)$ have been studied for the existence of stable solutions with inner transition layers. However, it seems that there exist few studies on the effect of $a(x)$ to this problem in the higher dimensional case.

In this paper, we consider the special case $a(x) = \chi_D(x)$ with a subdomain $D \subset \Omega$ and show existence of stable solutions with inner transition layers to

$$\epsilon^2 \Delta u + (a(x)^2 - u^2)u = 0, \quad x \in \Omega, \quad \frac{\partial u}{\partial n} = 0, \quad x \in \partial\Omega.$$

Assume that $D = D_1 \cup D_2, \overline{D_1} \cap \overline{D_2} = \emptyset, \overline{\partial D \cap \Omega} \subset \Omega$ and $\partial D_1, \partial D_2$ belong to the C^2 class. Then we have the following.

Theorem 1. *For sufficiently small $\epsilon > 0$, there exists a local minimizer u_ϵ of $J(u)$ on $H^1(\Omega)$ which has the following asymptotic behavior: u_ϵ converges to 1 uniformly on any compact subset of D_1, converges to -1 uniformly on any compact subset of D_2, and converges to 0 uniformly on any compact subset of $\Omega \setminus (\overline{D})$.*

Remark 1. The same result holds under the homogeneous Dirichlet boundary condition.

Remark 2. When D consists of several components, by choosing D_1 and D_2 suitably, Theorem says the existence of local minimizers which have different asymptotic behavior, i.e. are close to 1 on some components and are close to -1 on other components.

Remark 3. Although we think the smoothness of $\partial D_i, i = 1, 2$, is not necessary, we need at least C^2 regularity from a technical reason.

2 Useful Lemmas

We recall two useful lemmas.

Lemma 1 (Asymptotic behavior). *Let $D = \{x \in \mathbf{R}^n \mid |x| < \delta\}$, $g \in C^1(\mathbf{R}^1)$, and there exists a constant $T > 0$ such that $g(t) > 0$ $(t < 0)$, $g(T) = 0, g(t) < 0$ $(t > T)$. Suppose that $G(t) = \int_0^t g(s)\, ds$ has a unique maximum at $t = T$. Then, for a minimizer $u_\epsilon \in H_0^1(D)$ of*

$$\inf\{J_\epsilon(u; D) \mid u \in H_0^1(D)\},$$

where

$$J_\epsilon(u; D) = \frac{\epsilon^2}{2} \int_D |\nabla u|^2\, dx - \int_D G(u)\, dx,$$

we have $0 \le u_\epsilon(x) \le T$, $(x \in D), u_\epsilon(x) = u_\epsilon(|x|)$. Moreover, $u_\epsilon(x)$ converges to T uniformly on any compact subset $K \subset D$.

Next, let $g_1(x,t), g_2(x,t)$ be C^1-functions with respect to t and let

$$G_i(x,t) = \int_0^t g_i(x,s)\,ds, i = 1,2.$$

For $\eta_i \in H^1(D), i = 1,2$, consider the minimizing problem:

$$\inf\{J_i(u;D) \mid u - \eta_i \in H_0^1(D)\}, \quad J_i(u;D) = \frac{\epsilon^2}{2}\int_D |\nabla u|^2\,dx - \int_D G_i(x,u)\,dx.$$

Lemma 2 (Energy comparison). *$u_i \in H^1(D), i = 1,2$ be minimizers to the minimization problem above. Assume that there exit constants $m < M$ such that*

(a) $m \le u_i(x) \le M$ for $i = 1,2, x \in D$.

(b) $g_1(x,t) \ge g_2(x,t)$ for $x \in D, t \in [m,M]$.

(c) $\eta_1(x) \ge \eta_2(x)$ for $x \in D$.

Suppose $\eta_j \in C(\overline{D})$, $\eta_1(x) \not\equiv \eta_2(x)$ on ∂D. Then, we have $u_1(x) \ge u_2(x), x \in D$.

Although the proofs of these lemmas are known (see [3], [14]), we present it for reader's convenience.

Proof of Lemma 1. u_ϵ satisfies

$$\begin{cases} -\epsilon^2 \Delta u = g(u), & \text{for } x \in D = \{x \mid |x| < \delta\}), \\ u = 0, \text{ on } \partial D. \end{cases}$$

By the maximum principle and the condition on $g(t)$, we have $0 \le u_\epsilon(x) \le T$, $x \in D$. Gidas-Ni-Nirenberg's theorem implies

$$u_\epsilon(x) = u_\epsilon(|x|),\ u_\epsilon'(r) < 0,\ (r = |x| > 0).$$

For sufficiently small $\epsilon > 0$, define $w_\epsilon \in H_0^1(D)$ as follows:

$$w_\epsilon(x) = \begin{cases} T, & (|x| \le \delta - \epsilon) \\ -\frac{T}{\epsilon}(|x| - \delta), & (\delta - \epsilon < |x| \le \delta). \end{cases}$$

Since u_ϵ is a minimizer,

$$-\int_D G(u_\epsilon)\,dx \le J_\epsilon(u_\epsilon;D) \le J_\epsilon(w_\epsilon;D).$$

There exists a constant C_0 such that

$$J(w_\epsilon;D) \le \frac{\epsilon^2}{2}\int_{\{x|\delta-\epsilon<|x|\le\delta\}} |\nabla w_\epsilon|^2\,dx - G(T)|B(0,\delta)| + 2\max_{0\le t\le T}|G(t)||\{x|\delta-\epsilon<|x|\le\delta\}|$$
$$\le -G(T)|D| + C_0\epsilon.$$

where $|A|$ denotes the Lebesgue measure of a set $A \subset \mathbf{R}^n$. Thus

$$\int_D (G(T) - G(u_\epsilon))\, dx \le C_0 \epsilon.$$

Since $G(t)$ takes its maximum only at $t = T$, we have $G(T) - G(u_\epsilon) \ge 0$ on D.

Take arbitrary $r_0 \in (0, \delta)$ and fix. For $\sigma \in (0, \delta - r_0)$,

$$\begin{aligned}\int_D (G(T) - G(u_\epsilon))\, dx &\ge \int_{\{r_0 \le |x| \le r_0+\sigma\}} (G(T) - G(u_\epsilon)))\, dx \\ &= (G(T) - G(u_\epsilon(r_\epsilon)))|\{x | r_0 \le |x| \le r_0 + \sigma\}|\end{aligned}$$

holds for some $r_\epsilon \in (r_0, r_0 + \sigma)$.

Because the measure $|\{x|\ r_0 \le |x| \le r_0 + \sigma\}|$ is positive and independent of ϵ, as $\epsilon \to 0$ we have

$$0 \le G(T) - G(u_\epsilon(r_\epsilon)) \le C_1 \epsilon.$$

Since $G(t)$ takes its maximum only at $t = T$, we obtain $u_\epsilon(r_\epsilon) \to T$ as $\epsilon \to 0$. Noting $u_\epsilon(x) = u_\epsilon(|x|)$ and $u_\epsilon'(r) < 0$, we see

$$u_\epsilon(r_\epsilon) \le u_\epsilon(r) = u_\epsilon(|x|) \le T, \quad r = |x| \le r_0 \le r_\epsilon.$$

In particular, it follows

$$\max_{\{x|\ |x| \le r_0\}} |u_\epsilon(x) - T| \to 0 \text{ as } \epsilon \to 0.$$

By using a compactness argument, $u_\epsilon(x)$ converges to T uniformly on any compact subset of D.

Proof of Lemma 2. Let $M = \{x \in D |\ u_2(x) > u_1(x)\}$. Assume $M \ne \emptyset$. Then $D \setminus M$ contains nonempty open set. Put $\phi(x) = (u_2 - u_1)^+$. Then $\phi \in H_0^1(D)$, $\phi \not\equiv 0$ on D, and $\phi(x) = 0$ on $D \setminus M$. Since u_1, u_2 are minimizers respectively,

$$\begin{aligned} 0 &\le J_1(u_1 + \phi) - J_1(u_1) \\ &= \frac{\epsilon^2}{2} \int_M (|\nabla(u_1 + \phi)|^2 - |\nabla u_1|^2)\, dx - \int_M \int_{u_1(x)}^{u_1(x)+\phi(x)} g_1(x, s)\, ds\, dx \\ &\le \frac{\epsilon^2}{2} \int_M (|\nabla(u_1 + \phi)|^2 - |\nabla u_1|^2)\, dx - \int_M \int_{u_1(x)}^{u_1(x)+\phi(x)} g_2(x, s)\, ds\, dx \\ &= J_2(u_2) - J_2(u_2 - \phi) \le 0. \end{aligned}$$

This means that $u_1 + \phi$ is also a minimizer of J_1, and hence

$$-\epsilon^2 \Delta(u_1 + \phi) = g_1(x, u_1 + \phi).$$

Therefore, there exists a bounded function $c(x)$ such that

$$-\epsilon^2 \Delta\phi = g_1(x, u_1 + \phi) - g_1(x, u_1) = c(x)\phi.$$

The maximum principle or the unique continuation property leads a contradiction. Thus we can conclude $M = \emptyset$.

3 Proof of Theorem 1

In this section we use the notation

$$J_\epsilon(u;G) = \frac{1}{2}\epsilon^2 \int_G |\nabla u(x)|^2\,dx + \frac{1}{4}\int_G (a(x)^2 - u(x)^2)^2\,dx$$

for $u \in H^1(G)$ with $G \subset \Omega$. Let $\underline{v}_\epsilon$ be a positive global minimizer of

$$\inf_{v \in H_0^1(D_1)} J_\epsilon(v; D_1).$$

Existence of $\underline{v}_\epsilon$ follows from the standard argument. Moreover, by the maximum principle we have $0 < \underline{v}_\epsilon(x) < 1$ on D_1. By Lemma 1, $\underline{v}_\epsilon(x)$ converges to 1 uniformly on any compact subset $K \subset D_1$. Let $\underline{w}_\epsilon$ be a negative global minimizer of

$$\inf_{v \in H_0^1(\Omega \setminus \overline{D_1})} J_\epsilon(v; \Omega \setminus \overline{D_1}).$$

By Lemma 1, $\underline{w}_\epsilon(x)$ converges to -1 uniformly on any compact subset $K \subset D_2$ and to 0 uniformly on any compact subset $K \subset \Omega \setminus \overline{D_1 \cup D_2}$. Define $\underline{u}_\epsilon \in H^1(\Omega)$ as follows:

$$\underline{u}_\epsilon(x) = \begin{cases} \underline{v}_\epsilon(x), & x \in D_1 \\ \underline{w}_\epsilon(x), & x \in \Omega \setminus \overline{D_1}. \end{cases}$$

Lemma 3. *Let ν be the outward unit normal vector on ∂D_1. Then there exist positive constants δ_0, C_0 independent of ϵ such that*

$$\frac{\partial \underline{v}_\epsilon}{\partial \nu}(x) \le -\delta_0, \ (x \in \partial D_1),$$

$$\frac{\partial \underline{w}_\epsilon}{\partial \nu}(x) \ge -C_0\epsilon, \ (x \in \partial D_1).$$

Proof. For simplicity, we assume $\overline{D} \subset \Omega$. Let $\underline{v}_{\epsilon_0}$ be a positive global minimizer of $\inf_{v \in H_0^1(D_1)} J_{\epsilon_0}(v; D_1)$. Then , it is easy to see that $\underline{v}_{\epsilon_0}$ is a subsolution of the equation with $\epsilon(< \epsilon_0)$ on D_1. Since $v \equiv 1$ is a supersolution and the uniqueness of a positive solution, we have

$$0 \le \underline{v}_{\epsilon_0}(x) \le \underline{v}_\epsilon(x) \le 1, \ (x \in D_1).$$

This implies

$$\frac{\partial \underline{v}_\epsilon}{\partial \nu}(x) \le \frac{\partial \underline{v}_{\epsilon_0}}{\partial \nu}(x) \le -\delta_0 < 0, \ (x \in \partial D_1).$$

Let $w = -\underline{w}_\epsilon > 0$ be a positive minimizer of

$$\inf_{v \in H_0^1(\Omega \setminus \overline{D_1})} J_\epsilon(v; \Omega \setminus \overline{D_1}).$$

It suffices to show

$$\frac{\partial w}{\partial \nu}(x) \le C_0\epsilon, \ (x \in \partial D_1),$$

where ν be the outward (from D_1) unit normal vector on ∂D_1.

Take a smooth domain $(\Omega \supset)\tilde{D}_1 \supset \overline{D_1}$ s.t. $\overline{\tilde{D}_1} \cap D_2 = \emptyset$. Let $\tilde{w}$ be a global minimizer of

$$\inf\{J_\epsilon(v; \tilde{D}_1 \setminus D_1);\ v \in H^1(\tilde{D}_1 \setminus D_1),\ v = 0 \text{ on } \partial D_1,\ v = 1, \text{ on } \partial\tilde{D}_1.\}.$$

By Lemma 2, we have

$$w(x) \leq \tilde{w}(x),\ (x \in \tilde{D}_1 \setminus D_1).$$

Since $\tilde{D}_1 \setminus D_1 \subset \Omega \setminus \overline{D}$, $\tilde{w}$ satisfies

$$\epsilon^2 \Delta \tilde{w} = \tilde{w}^3.$$

Let $W_\epsilon(x) = \epsilon^{-1}\tilde{w}(x)$. Then

$$\Delta W_\epsilon = W_\epsilon^3,\ x \in \tilde{D}_1 \setminus D_1,$$

$$W_\epsilon(x) = 0,\ (x \in \partial D_1), \quad W_\epsilon(x) = \frac{1}{\epsilon},\ (x \in \partial\tilde{D}_1).$$

It is well-known (e.g., [5], [1], [13] and the references therein) that under the assumption ∂D_1 and $\partial\tilde{D}_1$ are of C^2 class there exists a unique positive solution to

$$\Delta V_\infty = V_\infty^3,\ x \in \tilde{D}_1 \setminus D_1,$$

$$V_\infty(x) = 0,\ (x \in \partial D_1), \quad V_\infty(x) = +\infty,\ (x \in \partial\tilde{D}_1).$$

Moreover, by comparison's theorem (see, e.g. [4]) we have

$$W_\epsilon(x) \leq V_\infty(x),\ (x \in \tilde{D}_1 \setminus D_1).$$

Thus, we have

$$w(x) \leq \tilde{w}(x) = \epsilon W_\epsilon(x) \leq \epsilon V_\infty(x), (x \in \tilde{D}_1 \setminus D_1).$$

For any compact subset $K \subset \tilde{D}_1 \setminus D_1$, where K include a neighborhood of ∂D_1,

$$\frac{\partial w}{\partial \nu}(x) \leq \epsilon \frac{\partial V_\infty}{\partial \nu}(x) \leq \epsilon C_0,\ x \in K.$$

This completes the proof of Lemma 3.

As an easy consequence of Lemma 3, we have the following.

Proposition 1. *There exists a sufficiently small $\epsilon_0 > 0$ such that, $\underline{u}_\epsilon$ is a subsolution for $0 < \epsilon < \epsilon_0$.*

Proof. We show that

$$\int_\Omega \left(\epsilon^2 \nabla \underline{u}_\epsilon \cdot \nabla \phi - (a(x)^2 - \underline{u}_\epsilon^2)\underline{u}_\epsilon \phi\right) dx \leq 0$$

holds for any $\phi \in C_0^\infty(\Omega)$ with $\phi(x) \geq 0$ in Ω. Note that by the elliptic regularity theorem we have $\underline{v}_\epsilon \in W^{2,p}(D_1)$ for any $p > n$ and hence $\underline{v}_\epsilon \in C^1(\overline{D_1})$. Also we have $\underline{w}_\epsilon \in W^{2,p}(\Omega \setminus \overline{D_1})$ for any $p > n$ and hence $\underline{w}_\epsilon \in C^1(\overline{\Omega \setminus \overline{D_1}})$. Thus we obtain

$$
\begin{aligned}
& \int_\Omega \left(\epsilon^2 \nabla \underline{u}_\epsilon \cdot \nabla \phi - (a(x)^2 - \underline{u}_\epsilon^2)\underline{u}_\epsilon \phi\right) dx \\
= & \int_{D_1} \left(\epsilon^2 \nabla \underline{v}_\epsilon \cdot \nabla \phi - (a(x)^2 - \underline{v}_\epsilon^2)\underline{v}_\epsilon \phi\right) dx \\
+ & \int_{\Omega \setminus \overline{D_1}} \left(\epsilon^2 \nabla \underline{w}_\epsilon \cdot \nabla \phi - (a(x)^2 - \underline{w}_\epsilon^2)\underline{w}_\epsilon \phi\right) dx \\
= & \int_{\partial D_1} \epsilon^2 \frac{\partial \underline{v}_\epsilon}{\partial \nu} \phi \, dS - \int_{D_1} \left(\epsilon^2 \Delta \underline{v}_\epsilon \phi + (a(x)^2 - \underline{v}_\epsilon^2)\underline{v}_\epsilon \phi\right) dx \\
- & \int_{\partial D_1} \epsilon^2 \frac{\partial \underline{w}_\epsilon}{\partial \nu} \phi \, dS - \int_{\Omega \setminus \overline{D_1}} \left(\epsilon^2 \Delta \underline{w}_\epsilon \phi + (a(x)^2 - \underline{w}_\epsilon^2)\underline{w}_\epsilon \phi\right) dx \\
= & \ \epsilon^2 \int_{\partial D_1} \left(\frac{\partial \underline{v}_\epsilon}{\partial \nu} - \frac{\partial \underline{w}_\epsilon}{\partial \nu}\right) \phi \, dS \\
\leq & \ \epsilon^2 \int_{\partial D_1} (-\delta_0 + C_0 \epsilon) \phi \, dS \leq 0.
\end{aligned}
$$

This completes the proof of Proposition 1.

In a similar way, let $\overline{v}_\epsilon$ be a negative global minimizer of

$$\inf_{v \in H_0^1(D_2)} J_\epsilon(v; D_2).$$

Let $\overline{w}_\epsilon$ be a positive global minimizer of

$$\inf_{v \in H_0^1(\Omega \setminus \overline{D_2})} J_\epsilon(v; \Omega \setminus \overline{D_2}).$$

Define $\overline{u}_\epsilon \in H^1(\Omega)$ as follows:

$$\overline{u}_\epsilon(x) = \begin{cases} \overline{v}_\epsilon(x), & x \in D_2 \\ \overline{w}_\epsilon(x), & x \in \Omega \setminus \overline{D_2}. \end{cases}$$

Then we have the following lemma which can be proved in the same way as in the proof of Lemma 3.

Lemma 4. *Let ν be the outward unit normal vector on ∂D_2. Then there exist positive constants δ_1, C_1 independent of ϵ such that*

$$\frac{\partial \overline{v}_\epsilon}{\partial \nu}(x) \geq \delta_1, \ (x \in \partial D_2),$$

$$\frac{\partial \overline{w}_\epsilon}{\partial \nu}(x) \leq C_1 \epsilon, \ (x \in \partial D_2).$$

By Lemma 4, we have the following proposition as in the proof of Proposition 1.

Proposition 2. *There exists a sufficiently small* $\epsilon_0 > 0$ *such that,* $\overline{u}_\epsilon$ *is a supersolution for* $0 < \epsilon < \epsilon_0$.

The following lemma is a consequence of the energy comparison lemma.

Lemma 5. *For* $0 < \epsilon < \epsilon_0$*, we have* $\overline{u}_\epsilon(x) > \underline{u}_\epsilon(x)$, $(x \in \Omega)$.

Proof. By using Lemma 2, we have $\overline{w}_\epsilon(x) \geq \underline{v}_\epsilon(x)$ on D_1. Moreover, by the strong maximum principle we have $\overline{w}_\epsilon(x) > \underline{v}_\epsilon(x)$ on D_1. In a similar way, we have $\overline{v}_\epsilon(x) > \underline{w}_\epsilon(x)$ on D_2. By the construction, these yield the desired result.

Proof of Theorem 1. By Lemma 5 and Brezis-Nirenberg's argument (see e.g. [2], [7], [12]), we have a local mimimizer u_ϵ of $J_\epsilon(u;\Omega)$ on $H^1(\Omega)$ such that

$$\overline{u}_\epsilon(x) \geq u_\epsilon(x) \geq \underline{u}_\epsilon(x), \ (x \in \Omega).$$

The asymptotic behavior of u_ϵ follows from the constructions of $\overline{u}_\epsilon, \underline{u}_\epsilon$, Lemma 1 and the proof of Lemma 3.

4 Some Extensions and Questions

In this section we discuss about possible extensions and open questions. First, for a given positive function $b(x)$, when $a(x) = b(x)\chi_D(x)$ with the same assumptions on D as in Theorem 1, we have a similar result. Moreover, it is possible to extend our result for the equation on compact manifolds, since the proof of Theorem 1 depends on simple minimizing problems, a comparison theorem and a solvability of solutions which blow up at the boundary.

Finally, we mention that the following questions remain open.

1. For a technical reason, in Theorem 1 we assume the condition $\overline{\partial D \cap \Omega} \subset \Omega$. It is an open question to show the same statement as in Theorem 1 for the case that $\overline{\partial D \cap \Omega}$ intersects $\partial\Omega$.

2. When $a(x) = \chi_D$, $\overline{D} \subset \Omega$, and D is a dumbbell like domain with a thin channel, can one still have a stable solution with inner transition layers? (cf. [2])

3. Without the smallness of $\epsilon > 0$, under certain assumption on D as in Theorem 1, can one show the existence of solutions which change sign?

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